# Data Compression in Recursive Estimation with Applications to Navigation Systems

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**The goal of data compression or preprocessing is to reduce computational requirements in a Kalman filter while retaining adequate estimation accuracy. A technique which averages batches of data is considered here. Guidelines are given for averaging, such as, how often to sample the data, when to average, and how many data points to average. The averaging technique is applied to aircraft navigation using two DMEs and either air data or inertial data. The results demonstrate that significant computation time reductions are possible with some covariance degradation. VOR/DME flight data of aircraft landing approaches were used to verify the analysis and show agreement with the analytical predictions.**

## I. Introduction

IN many estimation problems measurement data are available either continuously or at very short discrete intervals. Because of the limitation on computational capacity, the Kalman filter updating may be done at intervals longer than the measurement intervals. The standard Kalman filter formulation accepts only a single measurement vector every cycle. This could result in an exclusion of a large amount of information. The objective of this research is to investigate "preprocessing," "data compression," or "prefiltering"‡ techniques so as to reduce the computational requirement of the prefilter-filter combination while causing little covariance degradation.

Several investigators have studied some form of preprocessing. Hemesath[1] reduced the order of the measurement vector by differencing the position measurement made by VOR/DME and the position obtained by integrating the air data. This form of preprocessing does not couple the preprocessed measurements with the process equation. For a polynomial class of problems, Bar-Shalom[2] maximized the amount of information obtained from the preprocessor. Gotz[3] did similar work and used discrete Legendre polynomials to compress the data. The disadvantage of the polynomial technique is twofold. The process noise is difficult to include and it is restricted to the cases where the dynamics may effectively be represented by a polynomial. Dyer and McReynolds[4] applied data compression to the nonlinear orbit-determination problem by the least-square parameter estimation procedure. Womble[5] has derived an optimal preprocessor for cases where the problem is ill-conditioned. In general, an optimal weighting preprocessor is computationally expensive.

In this research, preprocessing is accomplished by averaging a batch of measurements between sucessive Kalman filter updates. The idea of simple measurement averaging is not new. It has been proposed by Schmidt,[6] Weinberg,[7] Klementis,[8] and others, the main objective being to smooth noisy data such as Doppler radar data. Previous work generally assumes special situations such as, no process noise, polynomial plant, etc. Previous results for these special cases are consolidated and extended to a situation where the plant is completely general. Also, guidelines are provided for applications of the averaging technique to a general problem.

## II. Motivations for Averaging

The objectives of data preprocessing is to slow the filter update rate to save computation time. In order to slow the filter update rate, a batch of measurements can be processed together instead of processing each measurement individually as shown in Fig. 1.

Consider a general multistage discrete process with discrete measurements. The state vector $x_k$ is described by the difference equation

$$x_{k+1} = \Phi_{k+1,k}x_k + w_k, \quad k = 0, 1, 2, 3 \ldots \tag{1}$$

where $\Phi_{k+1,k}$ is a known $n \times n$ transition matrix and $w_k$ is the process noise vector. Let

$$x_0 = N[\hat{x}_0, P_0]^{\S} \tag{2}$$

$$w_k = N[0, Q_k] \tag{3}$$

$$E[w_k w_j^T] = 0 \qquad j \neq k \tag{4}$$

$$E[w_k x_0^T] = 0 \tag{5}$$

An $r$-dimensional measurement vector $z_k$ available at every stage is given by

$$z_k = H_k x_k + v_k, \quad k = 1, 2, 3, \ldots \tag{6}$$

where $H_k$ is a known $r \times n$ measurement distribution matrix and

$$v_k = N[0, R_k] \tag{7}$$

A batch of $p$-measurements $z_{1,p} = \{z_1, z_2, \ldots, z_p\}$ can be used to obtain the maximum likelihood estimate of the state $x_p$ at the end of the batch. For simplicity, the sub-

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‡The terms data compression, preprocessing or prefiltering are used interchangeably in this paper.

§The notation $N[\hat{x}_0, P_0]$ in Eq. (2) above means that $x_0$ is normally distributed with mean $\hat{x}_0$ and covariance $P_0$.

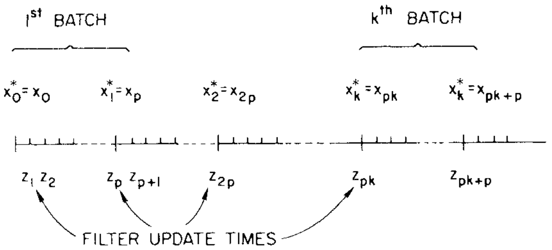


**Fig. 1 Schematic of recursive batch processing.**

scripts for the first batch are used. For the $(k + 1)$th batch, replace $z_i$ by $z_{pk+i}$ and the equations here hold true. Each measurement of the batch $z_i$ is expressed in terms of the forward state $x_p$. Then

$$z_i = H_i \Phi_{i,p} x_p + v_p' \tag{8}$$

where

$$v_p' = N[0, R_p'] = v_i + H_i \sum_{j=1}^{p-i} \Phi_{i,p+1-j} w_{p-j} \tag{9}$$

Because of the presence of the process noise $w_k$, the process and measurements are correlated and all the measurements in the same batch are correlated. The optimal processing algorithm requires an inverse of an $rp \times rp$ matrix which is generally very time consuming unless $p$ and $r$ are small.[9] A computationally feasible form for the processing algorithm may be obtained if the process noise is small enough so that the correlation it produces may be neglected.[10] Neglecting the correlation, the maximum likelihood filter equations are given by

$$\hat{x}_p = \bar{x}_p + P_p \sum_{i=1}^{p} \Phi_{i,p}{}^T H_i{}^T R_i'^{-1} [z_i - H_i \Phi_{i,p} \bar{x}_p] \tag{10}$$

where

$$P_p^{-1} = M_p^{-1} + \sum_{i=1}^{p} \Phi_{i,p}{}^T H_i{}^T R_i'^{-1} H_i \Phi_{i,p} \tag{11}$$

and $P_p$ and $M_p$ are defined as usual[11]:

$$M_p = E[(x_p - \bar{x}_p)(x_p - \bar{x}_p)^T] \tag{12}$$

$$P_p = E[(x_p - \hat{x}_p)(x_p - \hat{x}_p)^T] \tag{13}$$

For this algorithm the filter update requires more computations than the conventional Kalman filter because of the matrix multiplications $H_i \Phi_{i,p}$, etc. However, the algorithm saves $(p - 1)$ time updates. Equation (10) may be divided into preprocessor-processor equations as follows:

$$\hat{x}_p = \bar{x}_p + P_p [z_p{}^* - \bar{z}_p{}^*] \tag{14}$$

where the preprocessed measurement $z_p{}^*$ is given by

$$z_p{}^* = \sum_{i=1}^{p} \Phi_{i,p}{}^T H_i{}^T R_i'^{-1} z_i \tag{15}$$

Therefore, this preprocessor weights each measurement $z_i$ by the matrix weighting factor $\Phi_{i,p}{}^T H^T{}_i R_i'^{-1}$ and then averages. This form is complex and has been found to give computational savings by a factor of 2 or 3.

The Fisher information matrix is related to the inverse covariance matrix and is defined by Van Trees[12] and Maybeck.[13] Bar-Shalom[2] designed a preprocessor that preserves information in the sense of Fisher. Preserving all the information is computationally expensive. The objective here is to design a preprocessor that reduces the computational requirement while causing little loss of information. Equation (11) may be written as

$$I_T(p) = I_p(p) + \sum_{i=1}^{p} I_D(i/p) \tag{16}$$

where

$$I_T(p) = P_p^{-1} = \text{total (a posterior) information}$$

$$I_p(p) = M_p^{-1} = \text{a priori information} \tag{17}$$

$$I_D(i/p) = \Phi_{i,p}{}^T H_i{}^T R_i'^{-1} H_i \Phi_{i,p} = \text{information in measurement } z_i \text{ about the state } x_p$$

It can be seen that due to the dynamics and the process noise $I_D(i/p) \neq I_D(j/p)$. However, if the dynamics are slowly varying ($\Phi_i \simeq \Phi_j$) and the process noise is small compared to the measurement noise, i.e., $tr(H_i \Phi_i Q_i \Phi_i{}^T H_i{}^T) \ll tr(R_i)$, then the summation could be carried over the average information rate $I_D{}^*(p)$. One way to compute the average information rate is

$$pI_D{}^*(p) = p\left[\frac{1}{p}\sum_{i=1}^{p}\Phi_{i,p}{}^T H_i{}^T\right]\left[\frac{1}{p}\sum_{i=1}^{p} R_i'\right]^{-1}\left[\frac{1}{p}\sum_{i=1}^{p} H_i \Phi_{i,p}\right] \tag{18}$$

It can be verified that if the batch of measurements is averaged (i.e., equal weighting), then Eq. (18) gives the information in the averaged measurement. This "simple" averaging results in a loss of information. An expression for the information loss is given by Eq. (20). Defining

$$F_i = H_i \Phi_{i,p} \tag{19}$$

$$\sum_{i=1}^{p} I_D(i|p) - pI_D{}^*(p) = \sum_{i=1}^{p-1}\sum_{j=i+1}^{p} (R_i'^{-1}F_i - R_j'^{-1}F_j)^T \times R_j' \left[\sum_{l=1}^{p} R_l'\right]^{-1} R_i' \, (R_i'^{-1}F_i - R_j'^{-1}F_j) \tag{20}$$

Thus, in general, there is an information loss due to simple averaging unless $R_i'^{-1} H_i \Phi_{i,p}$ is a constant matrix and this loss is minimized if $R_i'$, $\Phi_{i,p}$, $H_i$ are slowly varying matrices. Note from Eqs. (7) and (9) that $R_i'$ is different than $R_i$ because of the process noise. In the filter Eqs. (10) and (11), the process noise was assumed to be small, and hence correlation due to process noise was neglected. It can be seen from Eq. (9) that if $R_i'$ is slowly varying, the process noise must be small. If, in Eq. (11)

$$I_p(p) >> pI_D{}^*(p) \tag{21}$$

i.e., if the a priori information about $x_p$ is large compared to the new information in the data batch, then $I_T(p)$ is less sensitive to the information loss due to averaging. Equation (21) is generally true after several data batches have been included into the filter.

## III. Averaging Algorithms

In the previous section it was shown that, under certain conditions, a simple average of a batch of data is an attractive way to preprocess. The best estimate of the state, given the constraint of simple averaging, will now be developed. It will be referred to as "exact" averaging.

In order to remove the lag due to averaging, measurements which are averaged together are referred to a fixed epoch of the state vector in the averaging interval. Weinberg[7] refers them to the end of the interval, while Bar-Shalom[2] refers them to the midpoint. We studied referring the averaged measurements to the beginning, the end, and the midpoint. For the exact algorithm, the beginning was preferred to the other points primarily from a computational standpoint. However, with simplifications discussed later in this section, the differences in the algorithms are not significant. The beginning-of-the-interval measurement (BOTIM) algorithm is presented here.

The equation for the averaged measurement referred to the state at the beginning of the interval becomes

$$z_k^* = \frac{1}{p}\sum_{i=1}^{p} z_{pk+i} = H_k^* x_k^* + v_k^* \tag{22}$$

where

$$H_k^* = \frac{1}{p}\sum_{i=1}^{p} H_{pk+i}\Phi_{pk+i,pk}, \quad x_k^* = x_{pk} \tag{23}$$

$$v_k^* = \frac{1}{p}\sum_{i=1}^{p}\left\{H_{pk+i}\sum_{l=1}^{i}\Phi_{pk+i,pk+l}w_{pk-1+l} + v_{pk+i}\right\} \tag{24}$$

and * denotes an averaged quantity. As usual

$$v_k^* = N[0, R_k^*] \tag{25}$$

The averaged measurement noise is correlated with the process noise and the correlation matrix is defined by

$$E[w_k^* v_k^{*T}] = S_k^* \tag{26}$$

Now the process equation becomes

$$x_{k+1}^* = \Phi_{k+1,k}^* x_k^* + w_k^* \tag{27}$$

where

$$w_k^* = N[0, Q_k^*] = \sum_{i=1}^{p}\Phi_{pk+p,pk+i}w_{pk+i-1} \tag{28}$$

and

$$\Phi_{k+1,k}^* = \Phi_{pk+p,pk} \tag{29}$$

It is seen that although $Q_k$ and $R_k$ are constant matrices, if $\Phi_k$ and $H_k$ are time varying, the $R_k^*$, $Q_k^*$, and $S_k^*$ are time varying. The process and averaged measurement equations are given by Eqs. (27) and (22). The filter equations may be obtained as in Ref. 11 by defining $D_k^* = S_k^* R_k^{*-1}$. The filter equations become

Time Update

$$\bar{x}_{k+1}^* = \Phi_{k+1,k}^*\hat{x}_k^* + D_k^*(z_k^* - H_k^*\hat{x}_k^*)$$
$$M_{k+1}^* = (\Phi_{k+1,k}^* - D_k^* H_k^*)P_k^*(\Phi_{k+1,k}^* - D_k^* H_k^*)^T + Q_k^* - D_k^* R_k^* D_k^{*T} \tag{30}$$

Measurement Update

$$\hat{x}_k^* = \bar{x}_k^* + K_k^*(z_k^* - H_k^*\bar{x}_k^*)$$
$$K_k^* = M_k^* H_k^{*T}(H_k^* M_k^* H_k^{*T} + R_k^*)^{-1}$$
$$P_k^* = M_k^* - K_k^* H_k^* M_k^* \tag{31}$$

The time update equation (30) is modified to take the correlation between the process noise and the averaged measurement noise into account, while the measurement update equation (31) is the same as the conventional Kalman filter equation. For this algorithm the measurement update always lags one filter cycle behind the most recent measurement. The BOTIM algorithm is thus a fixed-lag smoother-filter, the fixed lag being equal to one filter cycle. Also observe that,

$$P_k^* = P(t_{pk}|t_{pk+p}) \text{ and } \hat{x}^*(k) = \hat{x}(t_{pk}|t_{pk+p}) \tag{32}$$

$$M_k^* = P(t_{pk+p}|t_{pk+p}) \text{ and } \bar{x}^*(k) = \bar{x}(t_{pk+p}|t_{pk+p}) \tag{33}$$

i.e., the measurement update yields a smoothed estimate while the time update yields the current filtered estimate.

The BOTIM algorithm thus provides a viable fixed lag smoothing algorithm where real time and computationally inexpensive smoothing is desired, e.g., for on-line papameter estimation techniques, or for displays, etc. If some lag is acceptable or desired, then this algorithm has a large computational advantage over conventional fixed lag smoothing algorithms. From Eqs. (22-29), it is seen that several computations are required to evaluate the system and noise matrices $H^*$, $\Phi^*$, $R^*$, $Q^*$, and $S^*$. Procedures in the literature[10,14,15] were used to evaluate the computational requirements. If $\Phi_i$ and $H_i$ are slowly varying matrices and the process noise may be treated as stationary, then one or more of the modified noise matrices, $R^*$, $Q^*$, $S^*$, may be treated as constants without excessive loss of estimation accuracy. The computational requirement for the following four simplified averaging algorithms is studied.

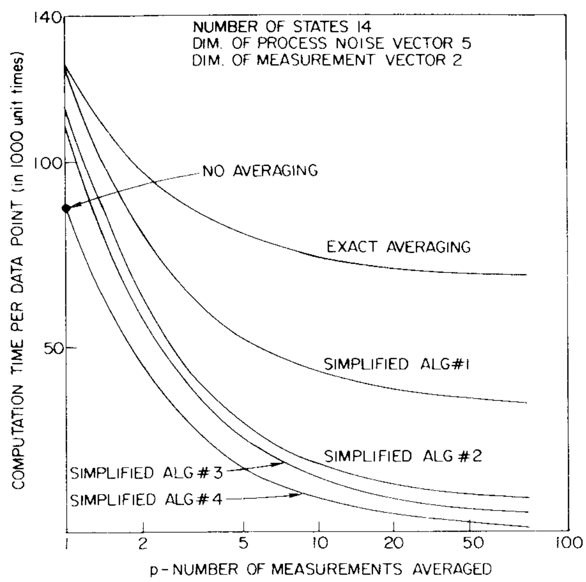


**Fig. 2 Computational requirement for the Kalman filter for aircraft navigation with two DMEs and air data.**

*Simplified Averaging No. 1:* $\Phi_i$ is assumed constant, then $\Phi_i^*$ and $Q_i^*$ are constants.

*Simplified Averaging No. 2:* $S^* = 0$ in addition to #1.

*Simplified Averaging No. 3:* $R_i^* = R/p$ in addition to #2.

*Simplified Averaging No. 4:* $H_i^* = H_i$ in addition to #3.

Figure 2 shows typical computational time requirements for a Kalman filter for aircraft navigation which is studied in Sec. V. With no measurement averaging (i.e., $p = 1$), the BOTIM algorithm requires more computations than the conventional Kalman filter. It should be remembered that the BOTIM algorithm obtains a smoothed estimate besides the filtered estimate. Although the BOTIM algorithm may not have much use in practice for $p = 1$, it is shown in these figures to emphasize the difference between the conventional Kalman filter and the BOTIM averaging algorithms. The simplified algorithms deliberately introduce "modeling errors" in the averaging filter, even when the models for the nonaveraging system are known perfectly. The "actual covariance"¶ of the averaging filter with modeling errors is larger than the covariance for the exact averaging algorithm which computes these matrices every cycle. Expressions for the covariance degradation due to these modeling errors are found in Ref. 10. The analysis is applied to a second-order model for (single axis) inertial navigation system and the results are given in Fig. 3. In this figure, contours of constant percent covariance increase (over the exact averaging covariance) in the position error are plotted for two simplifications. Simplifications 1–3 result in bounded errors while the simplification 4 results in divergence. It is interesting to note in this example that assuming $S_c^* = 0$ and $R_c^* = R/p$ results in a smaller degradation than just assuming $S_c^* = 0$. Two time varying filters are simulated in Sec. V for aircraft navigation using two DMEs and either air data or

¶ Because of modeling errors the actual state of the filter is different from the computed state. The covariance of the actual state is defined as the actual covariance.

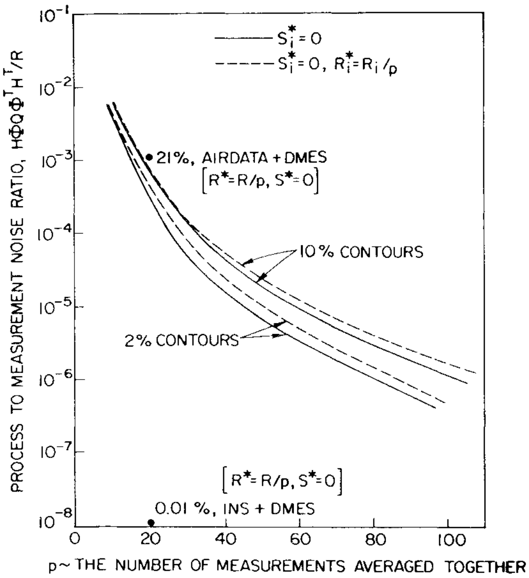


**Fig. 3 Contours of constant percent covariance degration due to simplifications in $R_i$* and $S_i$*.**

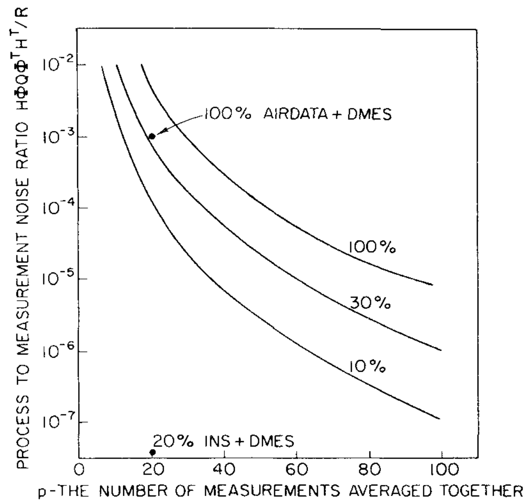


**Fig. 4 Contours of constant percent covariance increase due to exact averaging.**

inertial data. The results of these simulations are marked by dots (•) in Fig. 3. These two points match reasonably well with the contours drawn for the second-order model. It was observed that with $H_i^* =$ constant and the covariance of the exact algorithm in steady state, the actual variance was increasing with time, indicating that the filter was divergent. The rate of divergence was small, indicating that for short times the simplification $H_i^* = H_i$ was valid.

Figure 4 shows the percent covariance degradation over an optimal filter (no averaging) for the exact averaging of Eqs. (30) and (31). These results are based on the second-order system used to generate Fig. 3. The total degradation for a particular averaging algorithm can then be determined by using both Figs. 3 and 4.

## IV. Guidelines for Measurement Preprocessing

The guidelines presented here are intended to aid in the preliminary design of an averaging filter without solving the problem completely. The final design, as always, involves several iterations.

### How Often to Sample

Sensor data are typically analog, e.g., IMU, ILS, VOR, etc. For digital estimation, the continous data needed to be sampled and digitized. The desired sample rate is the slowest possible rate that does not result in "excessive" loss of information. If measurements are received from a piece of hardware at a given rate which is faster than necessary, the data may be ignored by the Kalman filter between samples. On the other hand, if the device has flexibility to vary the sample rate, then the rate should be matched with the necessary sample rate.

Not much is found in the literature on effects of sample rate in the estimation problem. Aström[16] has given a method for evaluation of covariance degradation due to sampling as compared to using continuous data. However, it is a complicated method as it needs a complete solution of the continuous estimation problem. A different approach is taken here.

Before the data is sampled it is typically passed through a low pass analog prefilter to prevent aliasing of high frequency noise components. The choice of the prefilter cutoff frequency should be high enough to eliminate any signal distortion but no higher than necessary because of the effect on the sample rate. For a perfectly bandlimited prefilter, the choice of sample rate would be clearly given by the Nyquist sample theorem, and the cutoff frequency of the analog prefilter would determine the sample frequency. (Since the signal can be accurately reconstructed if sampled at twice the cutoff frequency, any faster sampling would be wasteful.) For realizeable prefilters, the signal is not perfectly bandlimited. In this case, the choice of sample frequency is not as clear. A simple information approach with a first-order prefilter showed that for the ratio of sample interval ($\Delta$) to filter correlation time ($\tau$), $\Delta/\tau$, less than 0.25 the information in the sampled measurement is within 0.5% that of the continuous measurement. On the other hand, for $\Delta/\tau > 1$, the information loss is substantial (>7.6%). A good choice for $\Delta/\tau$ is thus $0.25 \leq \Delta/\tau \leq 1.0$ depening on how much information loss may be tolerated.

### When to Average

The measurement averaging technique is attractive if the conditions obtained earlier are satisfied. These conditions are: 1) that $\Phi_i$ and $H_i$ matrices be slowly varying, 2) that the process noise be smaller than the measurement noise, i.e., $tr(H_i\Phi_{i,p}Q_i\Phi_{i,p}{}^TH^T{}_i) \ll tr(R_i)$, and 3) that the a priori information about the state be much larger than the new information in the measurement batch [Eq. (21) satisfied].

The first two conditions insure that the loss of information be small, while the third condition insures that the estimation error be insensitive to the information loss due to averaging. If Condition 2 is satisfied, then after several data batches are included in the filter, Condition 3 is automatically satisfied. Condition 3 is thus a transient condition. Condition 1 is likely to be met where the measurement noise correlation time is much short compared to the system characteristic times.

### How Many Data Points to Average

The single most important factor influencing how many data points can be averaged is the process to measurement noise ratio. In addition, the total degradation (Figs. 3 and 4) is a function of the number of measurements averaged and how fast $\Phi_i$ and $H_i$ are varying. Figure 4 may, therefore, be used to determine how many data points to average for a tolerable degradation. Although Fig. 4 is drawn for a specific example, it gives a preliminary idea

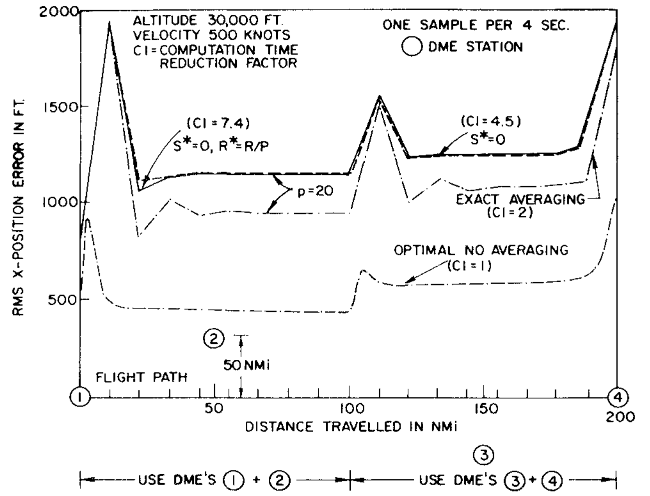


**Fig. 5 Simulation results for aircraft navigation with two DMEs and air data.**

of how much covariance degradation to expect for various $p$'s and $tr(H_i\Phi_{i,p}Q_i\Phi_{i,p}{}^TH^T{}_i)/tr(R_i)$. Unless $H_i$ and $\Phi_i$ are extremely slowly varying matrices so that $H_i{}^* - H_i \cong 0$, it can be seen from Fig. 2 that there is no computation savings beyond $p = 20$ to 25. If components of the measurement vector have significantly different covariances, then a different number of measurements may be averaged for each component and a sequential update procedure may be used.

## V. Applications of Averaging to Aircraft Navigation

The conventional Kalman filter design using VOR/DME and either air data or inertial data is studied by Bobick and Bryson.[17,18] They simulated the discrete equivalent of the continuous Kalman filter. Here, the same problems are considered. However, the filter mechanizations are assumed to be discrete and measurement data are averaged before introduction into the Kalman filter.

The air data and inertial systems represent two different process-to-measurement noise ratios $tr(H_i\Phi_{i,p}Q_i\Phi_{i,p}{}^TH^T)/tr(R_i)$. The ratios are $10^{-12}$ and $10^{-3}$ for the inertial and air data systems, respectively. The formulation of these problems is described by Bobick and, therefore, only the results are presented here. Bobick found that using two DMEs provided substantial accuracy improvements over

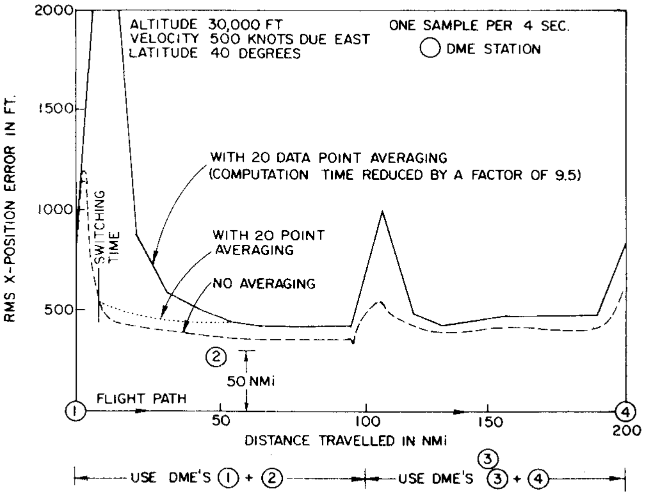


**Fig. 6 Simulation results for aircraft navigation with two DMEs and inertial data.**

**Table 1 Summary of error models**

| System | Model | Mean | rms Value | Correlation time |
|---|---|---|---|---|
| DME | random bias | 0 | 0.10 n mi | |
| | white noise | 0 | 0.07 n mi | 3.6 sec |
| Air data | exponentially correlated velocity error | 0 | 40 knots | 360 sec |
| INS | exponentially correlated gyro drift | 0 | 1°/hr | 5 hrs |
| | exponentially correlated accelerometer error | 0 | $10^{-4}\,g$ | 10 hrs |

using one VOR/DME for large (>20 naut mile) distances from navigation aids. Since more than one DME is generally available at jet altitudes, the simulation here uses two DMEs. An area navigation (RNAV) flight is considered here. The geometric relationship between the flight path and DME stations is depicted on the abscissa of Figs. 5 and 6. The INS, air data, and DME error models are summarized in Table 1. The correlation times are computed for an aircraft flying at a speed of 500 knots. The gyros are deliberately chosen coarse ($\epsilon$ = 1°/hr) because the flight duration is short.

DME measurements are sampled every four seconds and the discrete measurement noise is approximated as a purely random Gaussian sequence. The computational requirement curves of Fig. 2 shows that computations are not significantly reduced beyond $p = 20$, hence, twenty DME measurements are averaged together before introduction into the Kalman filter. Measurements are received every four seconds while the filter is updated every eighty seconds.

### Navigation with Two DMEs and Inertial Data

Here the aircraft is modeled kinematically.[10,18] There are six states, two positions, two velocities, and two DME biases. Four filters are simulated: one optimal with no averaging, one with exact averaging, and two with simplified averaging. One simplified algorithm assumes $S^* = 0$ and $R^* = R/p$. Figure 5 shows the simulation results for the $x$-position error.

During the initial transient, the condition of Eq. (21) is not satisfied. The position covariance is thus very sensitive to the loss of information due to averaging. After the initial transient has decayed, it is seen that the exact averaging filter has over 100% degradation over the optimal nonaveraging filter. Also, the simplified averaging filters have 20% degradation over the exact averaging filter. Both these results indicate that averaging 20 measurements result in a substantial information loss. This is consistent with the results in Figs. 3 and 4 for the process to measurement noise ratio of $10^{-3}$. For this problem, averaging 20 measurements results in a substantial (100%) error. If ten measurements are averaged, the total degradation may be reduced to 40 to 50% with the computations reduced by a factor of four over the conventional Kalman filter.

### Navigation with Two DMEs and Inertial Data

A 14th order model for the inertial navigation system is used.[10,17] It includes estimates of platform angle errors, gyro drifts, accelerometer biases, and DME biases. Figure 6 shows simulation results for the $x$-position error. As before, four filters are simulated: one optimal with no averaging, one exact with averaging, and two with simplified averaging. The dashed curve in Fig. 6 indicates the result for the conventional Kalman filter and is similar to that obtained by Bobick. For this filter, the measurements are received every four seconds and the filter is updated every four seconds. Since the process to measurement noise ratios is small, there is almost no degradation of covariance for the simplified averaging algorithms as compared to the exact averaging algorithm. Thus, in Fig. 6 the solid curve represents for all the three averaging algorithms. For this simulation, the IMU was assumed not to be finely aligned. During the in-flight alignment phase, the position error covariance increases. Because the condition of Eq. (21) is not satisfied initially, the position error covariance increases excessively with the averaging algorithm. This transient may be avoided either by ground alignment or by updating the filter every four seconds during the alignment phase of about 2 min and then switching to the averaging filter as shown by the dotted curve in Fig. 6. After the transient has decayed, the covariance degradation is only about 20%, while the computations are reduced by a factor of 9.5.

## VI. Verification with Flight Data

The analysis of previous sections is verified with flight data. Efforts were concentrated on 1) verification of the relationship between the correlation time of the noise and the sample rate and 2) verification of the covariance degradation due to averaging.

VOR/DME flight data of two-segment aircraft landing approaches were obtained from NASA Ames Research Center. For the same flights, radar position data were also available and were assumed to give the exact position of the aircraft. Denery[19] has described the flight program in detail. Figure 7 shows the flight schematic in the horizontal plane. The coordinate system used for filtering is shown in the figure. For simplicity, the data for the straight line flight segment (i.e., data between 24,000 and 16,000 ft horizontally from the touchdown) were used. As the nominal path is straight, a kinematic model is used for the aircraft. Because the measurement equations are nonlinear and the DME bias is significant compared to the distance of the aircraft to the DME station, the data was linearized about the current state estimate, i.e., the extended Kalman filter mechanization was used.

The filter estimate was compared with the smoothed radar measurement to obtain the estimation error. The average $x$-position error $\overline{\Delta x}$ is defined as

$$\overline{\Delta x}^2 = \frac{1}{n}\sum_{i=1}^{n}(\hat{x}_{i_{\text{filter}}} - x_{i_{\text{radar}}})^2$$

$\overline{\Delta y}$ and $\overline{\Delta z}$ are defined similarly and the performance index *(PI)* for the filter, which is used to compare various filters is defined as

$$PI = [\overline{\Delta x}^2 + \overline{\Delta y}^2 + \overline{\Delta z}^2]^{1/2}$$

In order to reduce the transient effect, a bias value of approximately 80% of the actual value was assumed by the filter. In an actual implementation of such a system, biases would not be available; however, typical navigation situations would yield much longer data spans than this special case. The intent in applying averaging to this data was to verify the algorithms with actual noise. The relatively short data spans of the noise abatement approaches were used because of their availability. Table 2 shows performance indices of optimal filters for various sample intervals. It can be seen that for sampling intervals of less than 0.64 sec, the performance index is not improved very much, while the performance index degrades rapidly as the sample interval is increased beyond 1.28 sec. Thus a sample interval between 0.64 and 1.28 sec is a good choice. An autocorrelation program implemented on VOR/DME data gave correlation times for VOR and DME of about 2.4 and 2.6 sec, respectively. Thus the choice of the sample interval of about half the correlation time of the noise is verified.

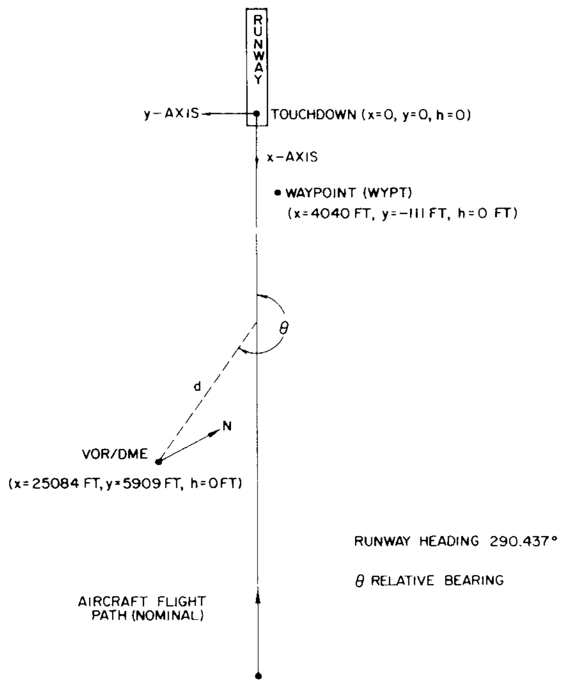


**Fig. 7 System mechanization in the horizontal plane.**

For the averaging filters, the sample interval of 1.28 sec was chosen. Table 3 shows the performance indices for averaging filters with different numbers of measurements averaged together. The table also gives the factors by which computations are reduced for various $p$'s. For the averaging filter simplifications $S^* = 0$ and $R^* = R/p$ were used and the averaged measurement was referred to the state at the end of the averaging interval. Figure 8 shows the improvement in the performance index due to averaging as compared to the filter that samples slower. Fig-

**Table 2 Effect of sampling rate on the performance index of the filter**

| Sampling interval (sec) | Performance index |
|---|---|
| 0.16 | 109 |
| 0.32 | 110 |
| 0.64 | 111 |
| 1.28 | 113 |
| 2.56 | 139 |
| 5.12 | 207 |
| 10.24 | 323 |
| 15.36 | 389 |

**Table 3 Comparison of filters with and without data compression**

| Filter update interval (sec) | Number of measurements averaged together | Computation time | Performance index (ft) |
|---|---|---|---|
| 1.28 | 1 | 1 | 113 |
| 2.56 | 2 | 1/1.5 | 115 |
| 5.12 | 4 | 1/2.7 | 125 |
| 10.24 | 8 | 1/4.5 | 153 |
| 15.36 | 12 | 1/5.7 | 240 |

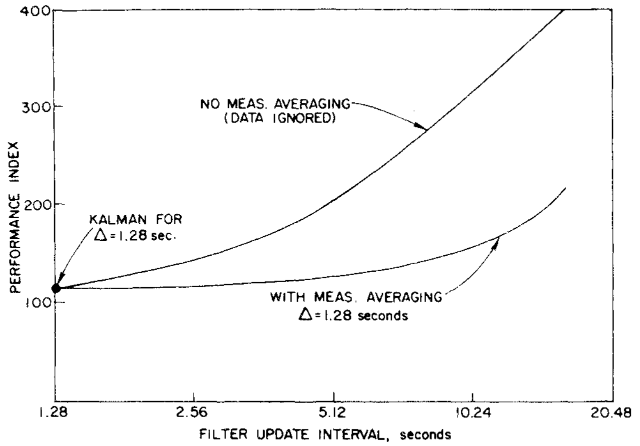


**Fig. 8 Comparison of performance index for averaging and nonaveraging filter.**

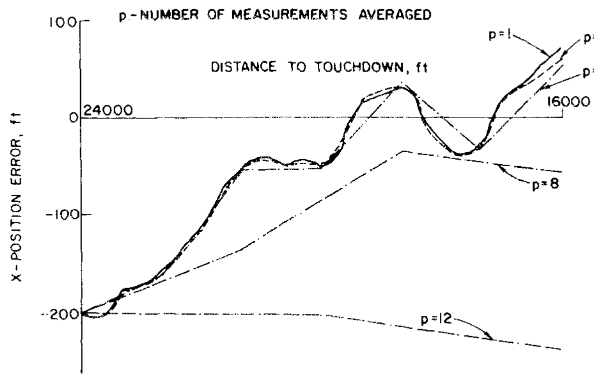


**Fig. 9 Time histories of $x$-position error for various $p$'s.**

ure 8 also shows how the performance degrades with averaging as compared to the filter that uses all the data without averaging. Figure 9 shows the time histories of the $x$ position error. From Figs. 8 and 9 it may be concluded that about 4 to 6 measurements may be averaged for a 10% degradation of the performance index. The process to measurement noise ratios for the VOR and DME are $10^{-2}$ and $0.5 \times 10^{-3}$, respectively. For the process to measurement noise ratio of $10^{-2}$, Fig. 4 indicates that about 4 to 6 measurements may be averaged, thus providing a verification of the analysis with flight data.

## VIII. Conclusions

This work demonstrated that data preprocessing or data compression results in a substantial computation savings with little covariance degradation for certain applications. An attractive way to compress data is to average the data between successive filter updates and modify the filter equations accordingly. When the process noise is present, the filter algorithm needs some simplifications to make the algorithm computationally attractive. If the conditions in Sec. II are met, then the covariance degradation due to averaging is assumed to be small. The data should be sampled at the slowest permissible rate given by the guidelines. For allowable covariance degradation, the process to measurement noise ratio determines how many data points to average. The results with the flight data verify the theoretical and computational analyses.

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